# DESIGN OF EDUCATIONAL APPLETS FOR INCREASING CHILDREN'S ABILITIES TO RECOGNIZE PATTERNS

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***Abstract:*** *The study is aimed to describe the process of creation (conception, design and implementation) of a collection of educational applets for the topic "repeating patterns" for children in preschool and young school age. The aim was to create applets which are simulating didactic tasks designed for finding a rule (core) in repeating patterns during pupils' game. Method Design Based Research was applied for development and implementation of interactive applets resulting in a collection of applets with grading tasks: a) identification of the pattern and ability to continue in the pattern (at different positions), b) finding a mistake in pattern and ability to correct it (using different ways), c) complete a missing sequence in the pattern (selecting from options). The applets are implemented through technologies html5, css3 and JavaScript without any additive browser plugins. The design and implementation reflects mathematical correctness and age suitableness as well as user comfort and simple schematic graphical design. The collection of applets is available on web page www.delmat.info (www.delmat.org).*

**Keywords:** applets, design, education, patterns, children.

## INTRODUCTION

One of the areas of the research project "Optimization of mathematics teaching materials based on analysis of the current needs and abilities of pupils of younger school age" is also a topic devoted to patterns and structures. The aim of the problem solving process is to design, create and verify such interactive and educational materials, environments respectively, in which children may discover the rules, patterns and structures on their own. In this paper we describe the factors that influenced production of high-level design educational applets focused on the problems devoted to patterns. During this process the methodology of the applet's elaboration was created which included requirements of the content expertise, design of functionality and visual adaptation of the applets. Based on these materials, software developers constructed low-level design while implementing the applets.

The author's *pedagogical content knowledge* is the condition for creating high quality didactical educational materials (applets included). The term didactical content knowledge was specified by Shulman (1986) and it integrates two factors: content knowledge and pedagogical knowledge. Slavík, Janík, Najvar and Knecht (2017) claim that didactical content knowledge is a common interpretational framework that connects theoretical didactics and practical didactics. It was important to apply both of these components while designing high-level design educational applets. In the following paragraphs we will try to clarify the basics of the didactical theoretical-practical principles about patterns that were implemented into the applet design.

## 1. THE CONTENT PRINCIPLES OF PATTERN'S APPLETS

Mathematicians often search and subsequently use patterns in the problem solving process. Steen (1998) stated that mathematics is a "*science of patterns*". He also argued that mathematics is a theory based on relations among patterns and applications of mathematics that arise from observation and pattern adaptation. The patterns can be found in arithmetic, algebraic and also in geometric structures. Development of informatics or statistical sciences also provides arguments for searching and developing rules and dependencies. The process of rule searching or relation searching is a process, in which the generalisation as a basic predisposition for the development of abstract thinking is often applied. At the same time, working with patterns helps to develop the ability to predict and detect principles in situations in which it is not clear at the first sign. Experiences with patterns allow us to identify a system, order, arrangement or principle. Therefore, it is important to develop an ability to work with patterns already in pre-school and young school age of children at appropriate level. In order to make the problem solving interesting, playful and compelling for children and mathematically and didactically correct at the same time we have decided to design and implement educational applets with an aim to develop abilities of children to recognise the pattern.

According to Clements and Sarama (2014) "*patterning is the search for mathematical regularities and structures*". Patterning is a process in which children expand not only their knowledge but also the way of thinking. In mathematics we work with different types of patterns whereas the pattern is being made up of elements (objects) and a rule. If the elements of a pattern are numbers, for example, we speak of number patterns; if the elements of pattern are pictures, we speak of image patterns and so on. The rule determines, for example, whether it is a repeating pattern in which the sequence of elements repeats itself or it is a growing pattern in which dynamic growth applies for some of the sequence's elements, perhaps even it is a recursive pattern in which the rule determines that following elements are made of the previous elements and so on. However, in this study we will only deal with fundamentals and the applications of repeating patterns.

A repeating pattern is a sequence of components that arises from a cyclically repeating core of pattern. The core of pattern is made of sequence of repeating components consisting of items (e.g. shapes, colours, numbers). The core must be clearly identifiable. Many patterns are created by a combination of different components but they can have the same fundamental essence. For example, a repeating number pattern (1, 2, 1, 2, 1, 2, ....) and a repeating shape pattern (triangle, circle, triangle, circle, triangle, circle, ...) or a repeating colour pattern (red, blue, red, blue, red, blue, …) are of the same type. In these cases the same rule of alternation of two components is applied. We mark this type of pattern by using symbols (ABABAB), therefore, the core has the length of two (AB). A different type of repeating pattern consists of two elements as well for example (ABBABBABB). In this pattern the core consists of a sequence (ABB) and its length is three. For a pre-school children or younger school aged children these two introduced samples are not equally demanding. The hierarchy of pattern complexity has been investigated by many authors (Orton (Eds.), 2005; Clemson and Clemson, 1994; Desli and Gaitaneri, 2017; Gadzichowski, 2012, Skoumpourdi, 2013) from many different perspectives considering different criteria. There is a consistency in all of the available research results claiming that the simplest linear repeating pattern for children is the type of pattern (ABABAB). Three years old and older children are able to identify this type of pattern (sometimes even earlier). The researchers try to define the appropriate degree of difficulty with consideration to cognitive theories. Therefore, it is evident that the scale of difficulty varies according to the author. Threlfall (in Orton (Eds.), 2005) forms a sequence of the tasks according to the level of difficulty of a repeating pattern originally designed by Vitz and Todd (1967): (AB), (AAABBB), (AABB), (AAB), (AAAB), (ABC), (AAABBBCCC), (AABBCC), (ACCCBCCC), (AAABC), (AABC), (AABBC).

The level of difficulty is not only dependent on the number of the objects and the length of the core of the objects but on the complexity of the rule as well. For example, from the sequence mentioned above, ABC, AABBCC and AAABC patterns contain three elements but are not of the same level of difficulty. The length of the core AABBCC is bigger than the length of the core in AAABC pattern, however, the shorter pattern is more difficult.

The level of difficulty is also influenced by environment and material that the pattern is made of. Our long-term observations showed differences between work with 3D objects, printed handouts and virtual objects. Children, who work with real objects (cubes, tokens and so on) and discover the pattern, have a tendency to continue without end. In other words, they often continue if there is a space to continue. However, children using printed handout continue only until there is no longer any empty space on the handout. From the point of view of rule discovery and ability to continue with the pattern discovery tasks in virtual space are the least open. This is partly due to production application rules and partly due to technical constrains. While discovering the rule automatic control is often a help to the problem solver. This way the child is not confronted by its own control, which can become a limiting factor. On the other hand, feedback about the success of the solver is a motivating satisfaction for the child.

Another factor of the pattern difficulty is formulation of the task. Previous characteristics of the difficulty were linked to the formulation of the given task such as: “observe and repeat”; “observe, find the rule and continue” that are the simplest. More difficult and more open at the same time is the task “find a mistake and correct it”. Also difficult but not because of the openness are tasks such as “fill in the missing element” or “choose, which sequence has to be completed”.

There are more types of tasks for the work with patterns. Also, there are more situations and environments in which activities with patterns can be implemented with a different character such as auditory or kinetic etc. As showed above, it is clear that there is a number of different possibilities of how to arrange a collection of tasks on repeating patterns. The most important ambition of ours was to achieve the consistency of our proposal of educational applets with current needs, abilities and interest of children of younger school age. Thus, the content structure of applets should be considering the needs of children from the cognitive and affective point of view. Moreover, it should be reflecting potentially different level of thinking of children.

## 2. TECHNICAL NOTES ON PATTERN’S APPLETS

Our requirements for designing the applets related to their technical solution were mainly based on the free accessibility of applets for mobile education, intuitive user-friendliness for younger children, and integrability into an existing project website (www.delmat.org, www.delmat.info). In brief, we will describe the characteristics of applets implementation in terms of the basic software principals.

The term “applet” was originally used in connection to Java Applets. Dynamic changes and development of the Internet browsers, especially HTML and CSS, gradually eliminate usage of input components such as Java or Flash respectively. That is because of their dependence on other technologies and because they are not usable on all, especially mobile, platforms. In this case the term applet was used purposefully as the work area of the task has clear boundaries on the web page with dedicated functionality and the remaining parts are shared and similar in most cases. The aims of the project required the wildest possible usability even on devices that limit the above-standard requirements for hardware and software equipment in the school environment.

The chosen software solution uses the newest versions CSS, HTML and JavaScript in both two and three-dimensional animations in a way that most of the browsers nowadays and in the future can accurately display all of the applets with the tasks without extensive code interventions. The exceptions are browsers of Microsoft because of the incompatible implementation of the World Wide Web Consortium (W3C) standards. Since it is not a native application, no installation/ update is required. The web portal can be continually complemented with new tasks and language mutations. The applets are optimised in terms of data transfers over the Internet. If the criteria changes the web page itself does not reload, however, the pictures (icons) change. The same rule applies when the browser size changes or the mobile device (tablet) rotates - the web page dynamically changes the visual input without downloading data from the Internet (responsive design). The applets were graphically and functionally unified for standard computers (monitor/mouse/trackpad), touch-screen devices that keep gestures, switches and some components depending on another principle (mouse clicks, mouse double clicks, right mouse button and so on.)

With regards to the target group of children in pre-school and younger school age, that often use mobile devices, applets use the "drag" & "drop" principle. Chosen control method is not only natural but also does not distract attention from the desired goal. Concentration retention is a subordinate design that is set minimalistic to schematic, without disturbing moments, extreme amounts of colour, components, and things such as "clickable" active elements. The aim of these attributes is to enable a child to concentrate for as long as possible on the mathematical essence of problem solving. In other words, we have tried to reduce any potential noise such as exaggerated animation or other disturbing phenomena that could possibly distract children.

## 3. METHODOLOGY OF THE RESEARCH

### 3.1 Research paradigm: educational design research

We have applied Design Based Research (DBR) to design, develop, and verify interactive learning applets. DBR uses qualitative, quantitative and mixed methods. If research is applied in education, we may speak of educational design research. According to McKenney and Reeves (2014) "*educational design research is a genre of research in which the iterative development of solutions to practical and complex educational problems provides the setting for scientific inquiry*". David (2007) claims that the aim of DBR is to uncover the relationships between the theory of education, proposed artefact and practice. This method of research is according to Anderson and Shattuck (2012) a practical method of research that can overcome the gap between theory and practice. The DBR method is based on a cyclical repetition of the intervention and its improvement until it is effective. This process is usually long-term, as confirmed by Pool and Laubscher (2016). There are several models and approaches to how DBR is implemented. The type of artefact, the conditions in which it will be verified can influence the choice of the model. Reinking and Bradly (2008) claimed that DBR is a suitable methodology for educational research and they focused on the questions concerning the use of intervention in education (e.g. how to use intervention in the classroom, what practical, methodological or ethical obstacles can arise, what is the potential of intervention etc.). Based on their analysis of existing DBR models and the underlying DBR theoretical framework in education, McKenney and Reeves (2012) created a generic model of education design research (Figure 1).

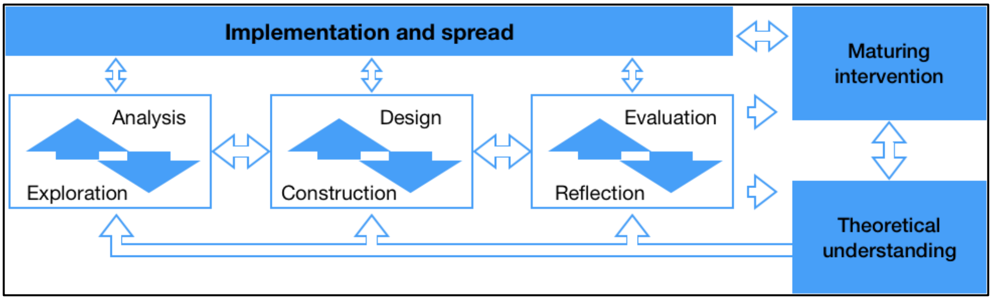



**Figure 1. Generic model of educational design research according to McKenney and Reeves (2012)**

*Source: Own work based on McKenney and Reeves (2012)*

The model consists of three main phases (analysis/exploration, design/construction and evaluation/reflection) arranged in a cyclic iterative structure that have a double reach: both theoretical and practical. This basic general model was most suited to our research goals, therefore became a paradigm for designing and verifying educational applets in order to increase a child's abilities to recognise patterns.

The method of educational research design enabled our prototype applications to be improved to their present form. Even the fact, that the development process has begun in 2008, it has not been completed yet and current applets are currently being verified. In the process of maturing intervention individual case studies are constantly being processes and important findings are selected to be included into new applet design.

### 3.2 The applet development process: I. phase of educational design research

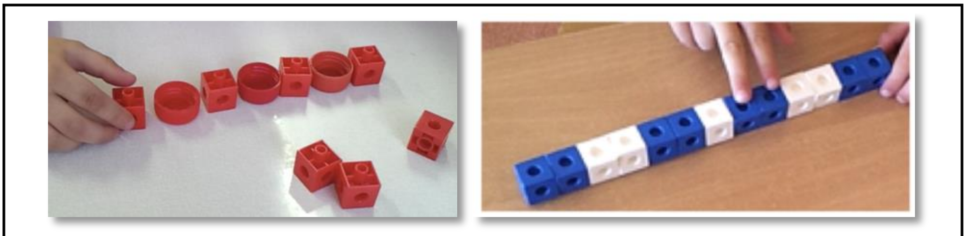


**Figure 2a, 2b. Exploration: a process of creation of patterns via manipulatives**

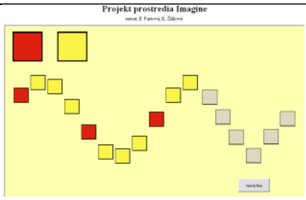
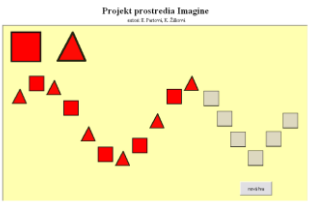
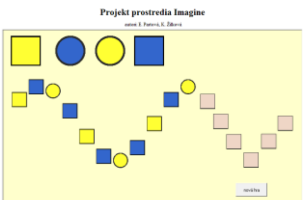
*Source: Own work*

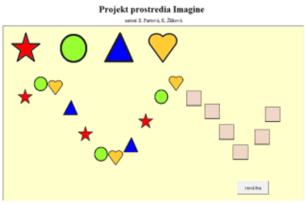
Based on our theoretical framework about repeating patterns (e.g. Orton, 2005; Clemson and Clemson, 1994) and also based on our multiple observations of pre-school children working with real objects (Figures 2a, 2b) we have made first four prototype applets (APP01 – APP04) in Imagine Logo environment in 2008. They were designed for pre-school and younger school aged children.

Imagine Logo is an object-oriented children's programming language controlled by events. The essence of designing applets was work with turtles, which are elementary objects of Logo. The linear repeating pattern was generated in applets using multiple turtles with assigned geometrical shape. The applets were controlled by the "drag and drop" method. Each applet used a different number of elements and colours, so the basis of each applet was unique. This approach enabled to verify not only the difficulty of individual patterns from the core of pattern points of view, but also from the recognition of colour and shape point of view. Significant attributes, which characterise prototype applets, and principles on which basis they were implemented, are shown in the table (Table 1).

**Table 1.**

**Summary of applets in Imagine Logo focused on the patterns**

| **Label** | **Graphical view** | **Types of objects** | **Rule of a pattern** | **Core of a pattern** |
|---|---|---|---|---|
| **APP01** |  | square,<br>triangle,<br>circle | ***1 shape 2 colours***<br>- pattern made of same shapes and alternation of two colours | *AB*<br>*ABB*<br>*ABBB*<br>*AABB* |
| **APP02** |  | square,<br>triangle,<br>circle | ***2 shapes 1 colour***<br>-pattern made of 1 colour and alternation of two shapes | *AB*<br>*ABB*<br>*ABBB*<br>*AABB* |
| **APP03** |  | square,<br>triangle,<br>circle | ***2 shapes 2 colours***<br>-pattern made of two shapes in two colours with random alternation of colour and shape | *AB, ABB,*<br>*ABBB,AABB,*<br>*ABC,*<br>*AABC, ABAC,*<br>*ABCA, ABBC,*<br>*ABCB, ACBB,* |

| | | | | |
|---|---|---|---|---|
| | | | | *ABCD,* |
| **APP04** |  | triangle,<br>circle,<br>heart<br>star | ***4 shapes 4 colours***<br>-pattern made of 4 shapes of different colours, each shape had constant colour | *AB, ABB*<br>*ABBB, AABB*<br>*ABC*<br>*AABC, ABAC*<br>*ABCA, ABBC*<br>*ABCB, ACBB*<br>*ABCD* |

*Source: Own work*

In applets it was not possible to set the level of difficulty of tasks according to the type of pattern. The first four elements of the pattern were generated randomly and the rest part of the pattern was their repetition. Therefore, it was not possible to modify the choice of task to the abilities of a child. The only way to do that was to generate another task and the teacher has to decide whether the task is appropriate for the child or not. We did not assume individual work of children with applets and we expected teachers' intervention not only in the choice of task but also in the exploration of a child's strategy through problem solving.

During verification of applets we identified some of the children's strategies and found out that the preference for shape or colour is not the same for all children. The difficulty of applets APP01 and APP02 was approximately on the same level. It was sufficient for children to follow only alternation in colours in APP01 (e.g. "red, yellow, yellow, red, yellow, yellow, ...") or in shapes in APP02 (e.g. "triangle, square, triangle, square, ..."). Solving tasks generated in APP03 was for children more difficult because they had to follow not only one criterium, but two criteria at the same time (e.g. "yellow square, blue square, yellow circle, yellow square, blue square, yellow circle, … "). In spite of the fact that the way patterns in applets were generated enabled to work with easier patterns applied in APP01 and APP02 also within APP03, for example, "yellow circle, blue square, yellow circle, blue, square, ...", which is a pattern with a core (AB). Applet APP04 enabled to generate all types of patterns as applet APP03 where the difference was only in the objects the pattern consisted of. Applets APP03 and APP04 were approximately on the same level of difficulty.

Most frequently computers and interactive boards were used in further verification of prototype applets APP01-APP04. At the same time new tasks were tested and we observed children as they were solving them, particularly via haptic manipulation (Figure 3) with real objects (Partová and Žilková, 2016a; 2016b). Results of our observations became the basis for input conditions for further analysis, preparation and creation of new applets.

**Figure 3. Working with patterns through manipulatives at the end of I. phase and beginning of II. phase of research**

*Source: Own work*

### 3.3 The applet development process: II. phase of educational design research

New analysis and design of applets were determined not only by educational (didactic) factors but also by technological changes and development of ICT means. The goal was to create applets that would enable mobile learning. At the same time we wanted to create not only more attractive environment but also care about the factor of concentration of a child on the content and the factor of elimination of potential disturbing elements. The result is collection of applets available on web page www.delmat.info (www.delmat.org).

New applets became prototypes for further research. It is a collection of five complex and content expanded applets (Figure 4) divided into three packages according to the type of task:

1. *Identification of the pattern and ability to continue in the pattern (at different positions)* – applet PAT1: http://www.delmat.info/a/3/;
2. *finding a mistake on pattern and ability to correct it (using different patterns)* – applets:
    a. PAT2 (by deleting an object): http://www.delmat.info/a/4a/,
    b. PAT3 (by covering an object with the right one): http://www.delmat.info/a/4b/;
    c. PAT4 (by inserting an object): http://www.delmat.info/a/4c/;
3. *complete missing sequence in the pattern (selecting from options)* – applet PAT5: http://www.delmat.info/a/5/.

The structure of applets enables to reflect the needs, abilities and interests of children on different levels that may be regulated by the choice of parameters of every task.

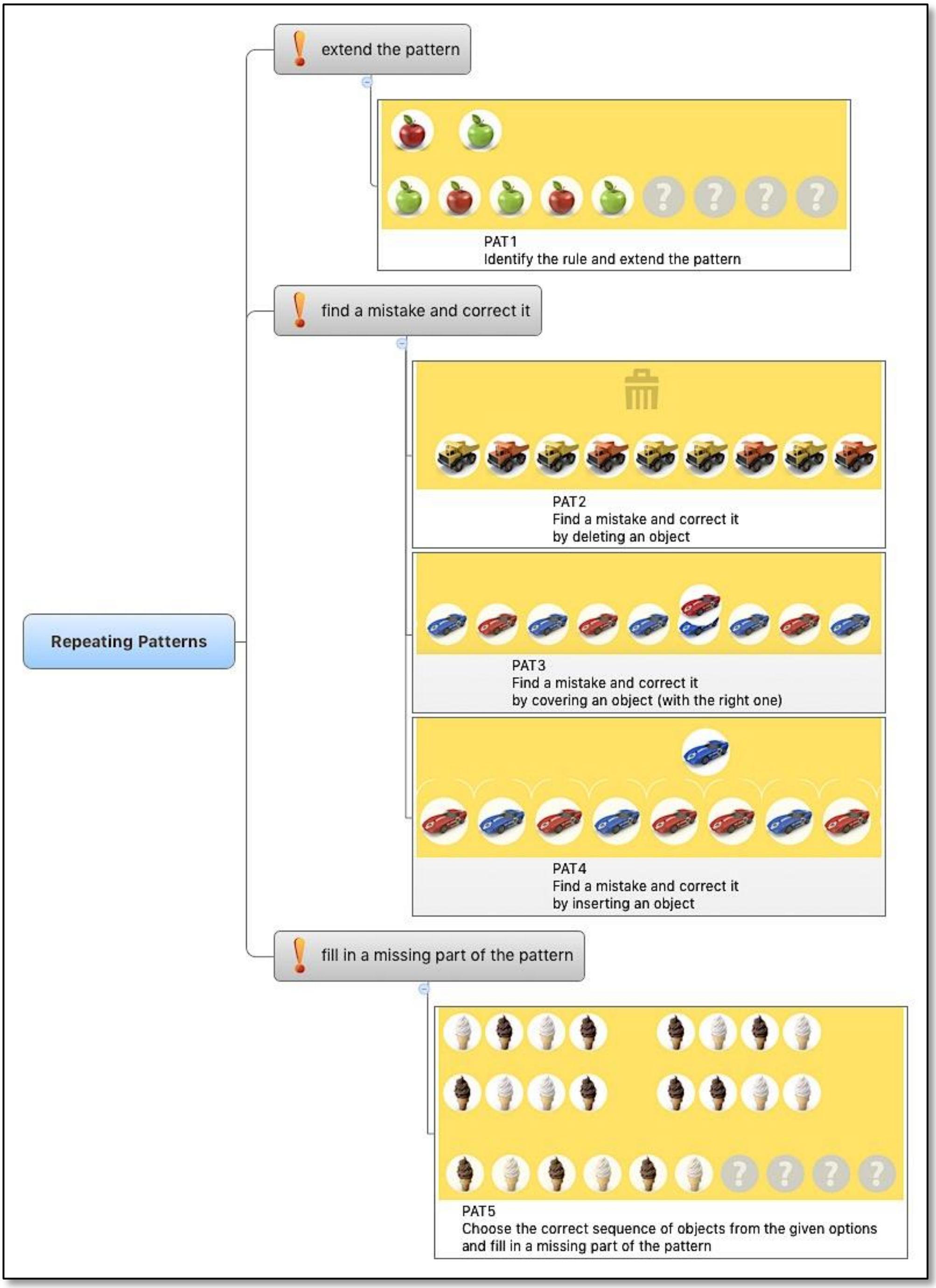



**Figure 4. Categorization of tasks in collection of applets: Repeating Patterns**

*Source: Own work*

A wide range of types of assignments, objects and complexity of a pattern eliminates limits of virtual environment presented above. The patterns are therefore generated according to the user predetermined parameters from which the common for all applets are:

- *difficulty of a pattern* – the pattern is generated according to the type of a pattern from these possibilities: AB, ABB, AABB, ABBB;
- *property (criteria) of a pattern* – the rule of alternation of elements in pattern is according to one of the properties: colour, shape, size;
- *objects of a pattern* – user may choose elements the pattern consists of: pictures of real objects, iconic (schematic) pictures, planar shapes, spatial shapes;
- *position of a pattern* – the pattern is generated horizontally or vertically;
- *form of a pattern* – the pattern may be displayed in a line or as a tilde.

Generation of tasks in applets enables to simulate relatively large amount of variable tasks. If we take into consideration, for example, only 4 possibilities of choice of difficulty of a pattern, 3 possibilities of choice of property and 4 possibilities of choice of object types then the applet may generate 48 different types of tasks. If we add to them another combination of other parameters, the number of tasks increases. Also for this reason we consider newly created applets more complex than the prototypes.

The phase of verification of applets is still in progress. However, based on the results already processed we identified some characteristic features connected with environment of applets. Respondents (children in preschool and young school age) worked with applets on different media: interactive board, laptop, tablet and smart phone.

In the sphere of technical manipulation with different media we observed a couple of curiosities. Working on an interactive board involves orientation on much greater area than a notebook, book or tablet. Despite that it was comfortable for the children. We observed the distance between the board and a child in the process of finding a rule and preference of digital pen over hand while moving the objects. Working on laptops did not require moving from the screen. Respondents manipulated with objects with the use of mouse or a finger (if the touch screen was available). In this case using finger appeared to be an easier way than using the mouse. Respondents who worked on tablets chose to work with a finger or digital pen according to their age that is probably caused by the development of fine motor skills.

In the cognitive area so far we have considered most significant the fact that our doubts with prompt feedback were not confirmed. Children identified easier patterns too soon, so they did not have to count on the trial and error method and they accepted feedback as confirmation of their work. In more complex patterns, where more possibilities of "answer" occurred, the trial and error method was more attractive but only for a short period of time. After that majority of children considered it a boring substitution and they started to find a rule.

In the area of a motivational impact we found out in all cases already processed that the applets aroused interest in children. It can be confirmed by their declarations, such as: "why don't we do it at school", "give us more such tasks".

The whole collection of applets (Figure 4) aimed at repeating patterns forms a research tool for quantitative and qualitative verification of their effectivity in terms of needs, interests and abilities of children. Besides verification of the existing applets we will focus on creation of more difficult applets in the next phases. Increase in difficulty will be ensured not only by an increase in the number of objects, length of the core of a pattern but also by increase in complexity of a rule according to the existing theoretical basis and published research papers.

## CONCLUSION

Educational design research does not have only an easy structure as was stated in the theoretical basis (Figure 1). McKenney a Reeves (2012) expanded their generic model by smaller cycles of different length within individual phases of research (analysis/exploration; design/construction; evaluation/reflection). These cycles are called micro-, meso- and macrocycles. The course of our research corresponded with this conception because it was necessary to verify results continuously within each cycle and each phase, correct changes based on verification, design and implement new elements again and so on. The process is spiral, it requires permanent analysis, constructions and evaluations. At the moment the phase of evaluation and reflection of new applets PAT1 – PAT5 is in process (Partová and Žilková, 2017a, 2017b, 2017c, 2017d, 2017e). We investigate the influence of the effectivity of applets on increase in the children's ability to discover the rule in a pattern and to apply it. By that we want to reflect needs of children in young school age in the context of development of their mathematical competences, generalization, the use of analogies, algorithmisation, finding relations and creating schemas in particular. At the same time we detect interests and preferences of children within individual applets in terms of chosen parameters of tasks. The goal is to achieve an inner motivation to solve children's problems with patterns and support their autonomy in problem solving. We also draw from the results of a study by Fox (2005) who regards activities with repeating patterns as productive learning occurrences. Results of the case studies they realized showed that "*children initiated activities that explored repeating patterns, pattern language, and the elements of linear patterns*". Based on our evaluation processes and individual case studies with children of young school age we assume that the interactive educational applets have potential not only to support inner motivation of children, but also increase in their abilities to recognize and to apply the pattern.

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